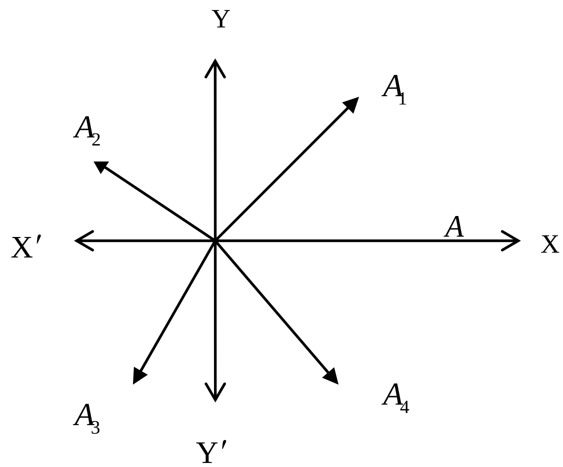
T1: Introduction: The word trigonometry is derived from Greek words trigon *meaning* a *triangle* and *metron* meaning *measurement*. In this branch of mathematics, we study relationship of sides and angles of triangle.

T2: Angle: It is defined as the amount of turn or rotation of a moving line with respect to a fixed line and fixed point.



Here OX is a fixed line and OA is a moving line. A_1, A_2, A_3 & A_4 are its positions as it is moving in anti-clockwise direction through angles $\theta_1, \theta_2, \theta_3$ & θ_4 .When it reaches again at point A, it completes one revolution and it is said to have described an angle of $360°$.

T3: Positive and Negative angle: When line OA is moving in anti-clockwise direction, then the angle described by it is said to **positive.** But when it moves in clockwise direction the angle made by it is said to be **negative.**

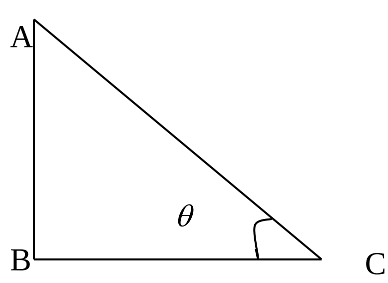
T4 Degrees Measure of an angle: A circle is divided into 360 equal parts and the angle subtended by each part is known as $1°$. Two perpendicular diameters of a circle divide it into 4 equal parts; therefore, each part subtends an angle of $90°$ at the centre.

$\therefore$ 1 right angle $= 90°$ (read as $90°$)

$1° = 60'$ (read as 60 minutes)

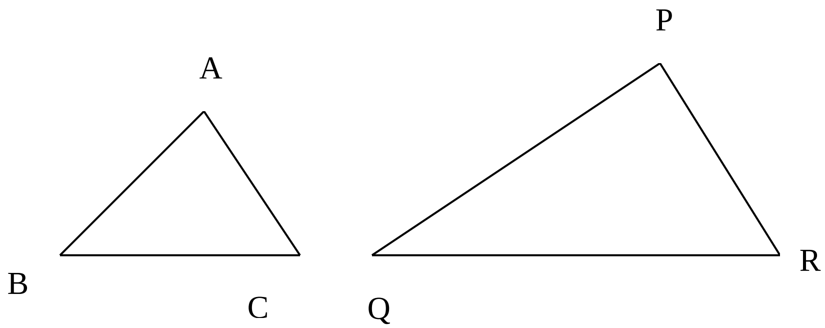
$1' = 60''$ (read as 60 seconds)

T5: Naming a Right angle:



Side Opp. Right Angle = Hypotenuse
Side Opp to Angle θ = Perpendicular
Side Adjacent to Angle θ = Base

T6: Property of Similar Triangl

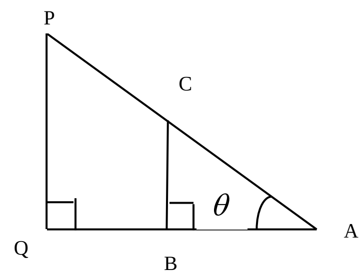


If $\Delta ABC \sim \Delta PQR$ then:

- $\dfrac{AB}{PQ} = \dfrac{BC}{QR} = \dfrac{AC}{PR}$
- $\angle A = \angle P, \angle B = \angle Q \;\&\; \angle C = \angle R$

T7: Trigonometric Ratios:

Consider a right angled triangle ABC and PQA in which $\angle B = 90° \;\&\; \angle Q = 90° \;\&\; \angle A = \theta$.



$\sin\theta = \dfrac{BC}{AC} = \dfrac{PQ}{PA} = \dfrac{Perpendicular}{Hypotenuse} = \dfrac{Side\,Opposite\,to\,angle\,\theta}{Hypotenuse}$
$\cos\theta = \dfrac{AB}{AC} = \dfrac{QA}{PA} = \dfrac{Base}{Hypotenuse} = \dfrac{Side\ Adjacent\ to\,\theta}{Hypotenuse}$
$\tan\theta = \dfrac{BC}{AB} = \dfrac{PQ}{QA} = \dfrac{Perpendicular}{Hypotenuse} = \dfrac{Side\,Opposite\,to\theta}{Side\ Adjacent\ to\,\theta}$
$\cos ec\theta = \dfrac{AC}{BC} = \dfrac{PA}{PQ} = \dfrac{Hypotenuse}{Perpendicular} = \dfrac{Hypotenuse}{Side\,Opposite\,to\,angle\,\theta} = \dfrac{1}{\sin\theta}$
$\sec\theta = \dfrac{AC}{AB} = \dfrac{PA}{QA} = \dfrac{Hypotenuse}{Base} = \dfrac{Hypotenuse}{Side\ Adjacent\ to\,\theta} = \dfrac{1}{\cos\theta}$
$\cot\theta = \dfrac{AB}{BC} = \dfrac{QA}{PQ} = \dfrac{Hypotenuse}{Perpendicular} = \dfrac{Side\ Adjacent\ to\,\theta}{Side\,Opposite\,to\theta} = \dfrac{1}{\tan\theta} = \dfrac{\cos\theta}{\sin\theta}$

T8 Trigonometrical Identities:

$\sin^2\theta + \cos^2\theta = 1$	$\sec^2\theta - \tan^2\theta = 1$	$\cos ec^2\theta - \cot^2\theta = 1$
$1 - \cos^2\theta = \sin^2\theta$	$\sec^2\theta - 1 = \tan^2\theta$	$\cos ec^2\theta - 1 = \cot^2\theta$
$1 - \sin^2\theta = \cos^2\theta$	$1 + \tan^2\theta = \sec^2\theta$	$1 + \cot^2\theta = \cos ec^2\theta$

T9 Trigonometric Ratios of Complementary Angles:

$\sin(90° - \theta) = \cos\theta$	$\cos ec(90° - \theta) = \sec\theta$
$\cos(90° - \theta) = \sin\theta$	$\sec(90° - \theta) = \cos ec\theta$
$\tan(90° - \theta) = \cot\theta$	$\cot(90° - \theta) = \tan\theta$

T10 Trigonometric Table:

Angle / T Ratio	0°	30°	45°	60°	90°
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
$\cos ec\theta$	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
$\sec\theta$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
$\cot\theta$	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

Trigonometric Identities (Questions)

Simplify:

1. $(\sec A - \tan A)(1 + \sin A)$
2. $\sin A(\text{cosec } A - \cot A)$
3. $\frac{1 + \cot^2 A}{1 + \tan^2 A}$

4. $\dfrac{\sin^3\theta-\cos^3\theta}{\sin\theta-\cos\theta}$
5. $\dfrac{1+\tan^3 A}{1+\tan A}$
6. $\dfrac{\sec^4 A-1}{\sec^2 A+1}$
7. $\dfrac{\sin^2\theta-\cos^2\theta}{\sin^4\theta-\cos^4\theta}$
8. $\dfrac{5+5\tan^2 A}{3+3\cot^2 A}$
9. $\dfrac{2-\tan\theta}{2\operatorname{cosec}\theta-\sec\theta}$
10. $\dfrac{5\operatorname{cosec}^2\theta-5\sec^2\theta}{\cot^2\theta-\tan^2\theta}$

Using trigonometric identities, write the following expressions as an integer:

11. $4\tan^2 A-4\sec^2 A$
12. $3\cot^2\theta-3\cos ec^2\theta$
13. $8\sin^2 A+8\cos^2 A$
14. $6\sec^2\theta-6\tan^2\theta+3$
15. $7\operatorname{cosec}^2 A-7\cot^2 A-5$

Prove the following identies

16. $\dfrac{\cos^2 A+\tan^2 A-1}{\sin^2 A}=\tan^2 A$
17. $\dfrac{\tan^2 A}{\tan^2 A-1}+\dfrac{\cos ec^2 A}{\sec^2 A-\operatorname{cosec}^2 A}=\dfrac{1}{\sin^2 A-\cos^2 A}$
18. $\dfrac{\sec\theta}{\sec\theta-1}+\dfrac{\sec\theta}{\sec\theta+1}=2\operatorname{cosec}^2\theta$
19. $\dfrac{\cos^2 A-\sin^2 A}{\sin A\cos A}=\cot A-\tan A$
20. $\dfrac{\sec^2\theta-\sin^2\theta}{\tan^2\theta}=\operatorname{cosec}^2\theta-\cos^2\theta$
21. $\dfrac{\cot^3 A-1}{\cot A-1}=\cos ec^2 A+\cot A$
22. $\sin^4\theta+\cos^4\theta=1-2\sin^2\theta\cos^2\theta$
23. $\sin^6\theta+\cos^6\theta=1-3\sin^2\theta\cos^2\theta$
24. $\dfrac{1-\sin^2 A}{\cos^2 A}=2\sec^2 A-1$

25. $\sqrt{\dfrac{1+\sin A}{1-\sin A}} = \sec A + \tan A$

26. $\sqrt{\dfrac{1+\sin A}{1-\sin A}} = \dfrac{\cos A}{1-\sin A}$

27. $\sqrt{\dfrac{1-\cos A}{1+\cos A}} = \dfrac{\sin A}{1+\cos A}$

28. $\dfrac{1+\sin A}{\cos A} + \dfrac{\cos A}{1+\sin A} = 2\sec A$

29. $\dfrac{\sin\theta-\cos\theta}{\sin\theta+\cos\theta} + \dfrac{\sin\theta+\cos\theta}{\sin\theta-\cos\theta} = \dfrac{2}{2\sin^2\theta-1}$

30. $(\sec\theta-\cos\theta)(\operatorname{cosec}\theta-\sin\theta) = \dfrac{1}{\tan\theta+\cot\theta}$

31. $\dfrac{2\cos^2 A-1}{\sin A\cos A} = \cot A - \tan A$

32. $\dfrac{\sec A+\tan A-1}{\tan A-\sec A+1} = \dfrac{1+\sin A}{\cos A}$

33. $\dfrac{\cot A}{1+\tan A} = \dfrac{\cot A-1}{2-\sec^2 A}$

34. $\dfrac{\tan\theta}{1-\cot\theta} + \dfrac{\cot\theta}{1-\tan\theta} = 1+\tan\theta+\cot\theta$

35. $\dfrac{\tan A}{\sec A-1} + \dfrac{\tan A}{\sec A+1} 2\operatorname{cosec} A$

36. $(1+\tan^2\theta)(1+\sin\theta)(1-\sin\theta) = 1$

37. $\tan\theta-\cot\theta = \dfrac{2\sin^2\theta-1}{\sin\theta\cos\theta}$

38. $(\sin\theta+\sec\theta)^2 + (\cos\theta+\operatorname{cosec}\theta)^2 = (1+\sec\theta\times\operatorname{cosec}\theta)^2$

39. $2\sec^2 A-\sec^4 A-2\operatorname{cosec}^2 A+\operatorname{cosec}^4 A = \cot^4 A-\tan^4 A$

40. $\dfrac{\sin\theta}{1+\cos\theta} + \dfrac{1+\cos\theta}{\sin\theta} = 2\operatorname{cosec}\theta$

41. $\dfrac{\sin\theta}{\cot\theta+\operatorname{cosec}\theta} = 2 + \dfrac{\sin\theta}{\cot\theta-\operatorname{cosec}\theta}$

42. $\tan^2\theta+\cot^2\theta+2 = \sec^2\theta\times\operatorname{cosec}^2\theta$

43. $\dfrac{1}{\operatorname{cosec} A-\cot A} - \dfrac{1}{\sin A} = \dfrac{1}{\sin A} - \dfrac{1}{\cos ecA+\cot A}$

44. $\dfrac{\cos^2\theta}{1-\tan\theta} + \dfrac{\sin^3\theta}{\sin\theta-\cos\theta} = 1+\sin\theta\cos\theta$

45. $\dfrac{\cos A}{1-\tan A} + \dfrac{\sin A}{1-\cot A} = \cos A+\sin A$

46. $(1+\tan A\tan B)^2+(\tan A-\tan B)^2=\sec^2 A\times\sec^2 B$

47. $\cot^2 A\left(\frac{\sec A-1}{1+\sin A}\right)+\sec^2 A\left(\frac{\sin A-1}{1+\sec A}\right)=0$

48. If $\tan A+\sin A=m$ and $\tan A-\sin A=m$; prove that $m^2-n^2=4\sqrt{mn}$

49. If $\cot\theta-\cos\theta=a$ and $\cot\theta+\cos\theta=b$; prove that $b^2-a^2=4\sqrt{ab}$

50. If $\cos\theta+\sin\theta=\sqrt{2}\cos\theta$, show that $\cos\theta-\sin\theta=\sqrt{2}\sin\theta$

51. If $\sin A+\cos A=p$ and $\sec A+\operatorname{cosec} A=q$ show that $q(p^2-1)=2p$.

52. If $\frac{\cos\alpha}{\cos\beta}=m$ and $\frac{\cos\alpha}{\sin\beta}=n$, show that $(n^2+m^2)\cos^2\beta=n^2$

53. If $x=r\sin A\cos C$, $y=r\sin A\sin C$ & $z=r\cos A$, prove that $r^2=x^2+y^2+z^2$

54. Prove that $(1-\sin\theta+\cos\theta)^2=2(1+\cos\theta)(1-\sin\theta)$

55. If $\sin\theta+\sin^2\theta=1$, prove that $\cos^2\theta+\cos^4\theta=1$

56. $\frac{(1+\sin\theta)^2+(1-\sin\theta)^2}{2\cos^2\theta}=\frac{1+\sin^2\theta}{1-\sin^2\theta}$

57. $\frac{\cos\theta}{\operatorname{cosec}\theta+1}+\frac{\cos\theta}{\operatorname{cosec}\theta-1}=2\tan\theta$

58. $\frac{\sin\theta}{\sec\theta-1}+\frac{\sin\theta}{\sec\theta+1}=2\cot\theta$

59. If $a\sec\theta+b\tan\theta=m$ & $a\tan\theta+b\sec\theta=n$, prove that $m^2-n^2=a^2-b^2$

Express in terms of Trigonometrical ratio of angles between $0°$ & $45°$:

1. $\sin 72°+\tan 72°$
2. $\cos 85°-\cot 63°$
3. $\tan 57°+\cot 75°$

Evaluate:

1. $\frac{\sin 20°}{\cos 70°}$
2. $\frac{\tan 42°}{\cot 48°}$
3. $\frac{\sec 80°}{\operatorname{cosec} 10°}$
4. $\sin 55°-\cos 35°$
5. $\tan 82°-\cot 8°$
6. $\operatorname{cosec} 36°-\sec 54°$
7. $3\sin 50°-3\cos 40°+\cos 0°$
8. $\sin^2 50°+\sin^2 40°$
9. $\frac{\cos^2 40°+\cos^2 50°}{\sin^2 38°+\sin^2 52°}$

10. $\left(\frac{\sin 35^\circ}{\cos 55^\circ}\right)^2 + \left(\frac{\sec 55^\circ}{\operatorname{cosec} 35^\circ}\right)^2$

11. $\sec^2 73^\circ - \cot^2 17^\circ$

12. $\operatorname{cosec}^2 20^\circ - \tan^2 70^\circ$

13. $\frac{\sin(90^\circ - \theta)}{\cos\theta} + \frac{\sin\theta}{\cos(90^\circ - \theta)}$

14. $\operatorname{cosec}^2 \theta - \tan^2(90^\circ - \theta)$

15. $\cot^2 \theta - \sec^2(90^\circ - \theta)$

16. $\frac{\cos 20^\circ}{\sin 70^\circ} + \frac{\cos\theta}{\sin(90^\circ - \theta)}$

17. $\sin A \cos(90^\circ - A) + \cos A \times \sin(90^\circ - A)$

18. $\sin A \sin(90^\circ - 1) - \cos A \cos(90^\circ - A)$

19. $\cot(90^\circ - A) \times \sin(90^\circ - A)$

20. $\frac{\cos(90^\circ - \theta) \times \cos\theta}{\tan\theta} + \cos^2(90^\circ - \theta)$

21. $\sin(90^\circ - A) \times \cos(90^\circ - A) = \frac{\tan A}{1 + \tan^2 A}$

22. $\frac{3\sin 52^\circ}{\cos 38^\circ} - \frac{\operatorname{cosec} 32^\circ}{\sec 58^\circ}$

23. $\tan 15^\circ \tan 20^\circ \tan 70^\circ \tan 75^\circ$

24. $\tan 15^\circ \tan 20^\circ \tan 45^\circ \tan 70^\circ \tan 75^\circ$

25. $\cot 10^\circ \cot 25^\circ \cot 65^\circ \cot 80^\circ$

26. $\frac{2\tan 53^\circ}{\cot 37^\circ} - \frac{\cot 80^\circ}{\tan 10^\circ}$

27. $\sin^2 20^\circ + \sin^2 70^\circ - \tan^2 45^\circ$

28. $\frac{\cos 70^\circ}{\sin 20^\circ} = \frac{\cos 59^\circ}{\sin 31^\circ} - \sin^2 30^\circ$

29. $\frac{\sin^2 20^\circ + \sin^2 70^\circ}{\cos^2 20^\circ + \cos^2 70^\circ} + \frac{\sin(90^\circ - \theta)\sin\theta}{\tan\theta} + \frac{\cos(90^\circ - \theta)}{\cot\theta}$

30. $\cos(40^\circ + \theta) - \sin(50^\circ - \theta) + \frac{\cos^2 40^\circ + \cos^2 50^\circ}{\sin^2 40^\circ + \sin^2 50^\circ}$

31. $\frac{\cos 70^\circ}{\sin 20^\circ} + \frac{\cos 55^\circ \cos ec 35^\circ}{\tan 5^\circ \tan 25^\circ \tan 45^\circ \tan 65^\circ \tan 85^\circ}$

32. $\frac{2}{3}(\cos^4 30^\circ - \sin^4 45^\circ) - 3(\sin^2 60^\circ - \sec^2 45^\circ) + \frac{1}{4}\cot^2 30^\circ$

33. $4(\sin^4 30^\circ + \cos^4 60^\circ) - \frac{2}{3}(\sin^2 60^\circ - \cos^2 45^\circ) + \frac{1}{2}\tan^2 60^\circ$

34. Find θ, if $\sin(\theta + 35^\circ) = \cos\theta$; where $(\theta + 36^\circ)$ is acute.

35. If $\sec 2\theta = \text{cosec}(\theta + 6°)$, where 2θ & $(\theta + 6°)$ both are acute; find the value of θ.

36. If $\tan 3\theta = \cot(\theta - 6°)$, where 3θ & $\theta - 6°$ are acute, find the value of θ.

37. If A & B are acute angles and $\tan A = \cot B$; prove that $A = B = 90°$

38. If $A + B = 90°$; show that $\cos A = \sin B$ & $\dfrac{\sec A}{\text{cosec}\, B} = 1$

39. If A, B & C are the interior angles of a triangle; show that: (i) $\sin\dfrac{A+B}{2} = \cos\dfrac{C}{2}$

(ii) $\tan\dfrac{B+C}{2} = \cot\dfrac{A}{2}$ (iii) $\sec\dfrac{C+A}{2} = \cos ec\dfrac{B}{2}$

40. If $\sin A + \text{cosec}\, A = 2$; show that; $\sin^2 A + \cos ec^2 A = 2$

41. If $\tan\theta - \cot\theta = 2$ show that: $\tan^2\theta + \cos^2\theta = 6$