



1. Show that $\left\{ i^{19} + \left(\frac{1}{i} \right)^{25} \right\}^2 = -4$
2. Show that $\left\{ i^{17} - \left(\frac{1}{i} \right)^{34} \right\}^2 = 2i$
3. Show that $i^n + i^{n+1} + i^{n+2} + i^{n+3} = 0, n \in N$
4. Evaluate $i^{37} + \frac{1}{i^{67}}$ (Ans: $2i$)
5. Evaluate $\left(i^{41} + \frac{1}{i^{257}} \right)^9$ (Ans: 0)
6. Show that $\frac{i^{592} + i^{590} + i^{588} + i^{586} + i^{584}}{i^{582} + i^{580} + i^{578} + i^{576} + i^{574}} = -1$
7. Evaluate $\sqrt{-25}$ (Ans: $5i$)
8. Evaluate $\sqrt{-225}$ (Ans: $15i$)
9. Evaluate $\sqrt{-36}\sqrt{-2}$ (Ans: $-6\sqrt{2}$)
10. Explain the fallacy: $1 = \sqrt{1} = \sqrt{-1 \times -1} = \sqrt{-1} \times \sqrt{-1} = (i)(i) = i^2 = -1$

Express the following (11-22) in $a + ib$ form

11. $(i)(2i) \left(-\frac{1}{8i} \right)^3$ (Ans: $0 + \frac{1}{256}i$)
12. i^{-39} (Ans: $0 + 1i$)
13. $(1-i)^4$ (Ans: $-4 + 0i$)
14. $\left\{ \left(\frac{1}{3} + \frac{7}{3}i \right) + \left(4 + \frac{1}{3}i \right) - \left(-\frac{4}{3} + i \right) \right\} \left(\text{Ans: } \frac{17}{3} + \frac{5}{3}i \right)$
15. $\left(\frac{1}{3} + 3i \right)^3$ (Ans: $-\frac{242}{27} - 26i$)
16. $(5-3i)^3$ (Ans: $(\sqrt{2}-6) + (\sqrt{3} + 2\sqrt{6})i$)
17. $\frac{1}{3+4i}$ (Ans: $\frac{3}{25} - \frac{4}{25}i$)



18. $\frac{(3-2i)(2+3i)}{(1+2i)(1-(2-i))} \left(\text{Ans: } \frac{63}{25} - \frac{16}{25}i \right)$
19. $\frac{1}{4-\sqrt{-5}} \left(\text{Ans: } \frac{4}{21} + \frac{\sqrt{5}}{21}i \right)$
20. $\left(\frac{1}{1-2i} + \frac{3}{1+i} \right) \left(\frac{3+4i}{2-4i} \right) \left(\text{Ans: } \frac{1}{4} + \frac{9}{4}i \right)$
21. $\frac{1}{1-\cos\theta+2i\sin\theta} \left(\text{Ans: } \left(\frac{1-\cos\theta}{2-2\cos\theta+3\sin^2\theta} \right) + \left(\frac{-2\sin\theta}{2-2\cos\theta+3\sin^2\theta} \right) i \right)$
22. $\frac{3-\sqrt{-16}}{1-\sqrt{-9}} \left(\text{Ans: } \frac{3}{2} + \frac{1}{2}i \right)$
23. Find the real value of x and y if $(1-i)x + (1+i)y = 1-3i$ (Ans: $x = 2, y = -1$)
24. Find the real value of x and y if $(x+iy)(2-3i) = 4+i$ (Ans: $x = \frac{5}{13}, y = \frac{14}{13}$)
25. If $a+ib = \frac{c+i}{c-i}$ where c is real, prove that $a^2+b^2 = 1$ and $\frac{b}{a} = \frac{2c}{c^2-1}$
26. $(u+iv)^{1/3} = x+iy$ $u, v, x, y \in R$. Show that $\frac{u}{x} + \frac{v}{y} = 4(x^2-y^2)$
27. Find the conjugate of $\frac{1}{3-4i} \left(\text{Ans: } \frac{3}{25} - \frac{4}{25}i \right)$
28. Find the conjugate of $\frac{(2+3i)^2}{2-i} \left(\text{Ans: } -\frac{22}{5} + \frac{19}{5}i \right)$
29. Find the real numbers x and y if $(x-iy)(3+5i)$ is the conjugate of $-6-24i$ (Ans: $x = 3, y = -3$)
30. Find the multiplicative inverse of $4+5i \left(\text{Ans: } \frac{4}{41} - \frac{5}{41}i \right)$
31. Find the multiplicative inverse of $(2+i)^2 \left(\text{Ans: } \frac{3}{25} - \frac{4}{25}i \right)$
32. If $\frac{a+ib}{c+id} = x+iy$ then prove that $\frac{a^2+b^2}{c^2+d^2} = x^2+y^2$



33. If $\frac{(a+i)^2}{2a-i} = p+iq$, show that: $p^2 + q^2 = \frac{(a^2+1)^2}{4q^2+1}$
34. If $a+ib = \frac{(x+i)^2}{2x^2+1}$, prove that $a^2 + b^2 = \frac{(x^2+1)^2}{(2x^2+1)^2}$
35. If $x+iy = \sqrt{\frac{a+ib}{c+id}}$, prove that: $(x^2+y^2)^2 = \frac{a^2+b^2}{c^2+d^2}$
36. If $(1+i)(1+2i)(1+3i)\dots(1+ni) = (x+iy)$ show that: $2.5.10\dots(1+n^2) = x^2 + y^2$
37. If $(a+ib)(c+id)(e+if)(g+ih) = A+iB$, prove that: $(a^2+b^2)(c^2+d^2)(e^2+f^2)(g^2+h^2) = A^2+B^2$
38. If $x = -5 + 2\sqrt{-4}$ find the value of $x^4 + 9x^3 + 35x^2 - x + 4$ (Ans: -160)
39. Find the value of $x^3 + 7x^2 - x + 16$, when $x = 1 + 2i$ (Ans: -17 + 24i)
40. If z is a complex number such that $|z| = 1$ prove that $\left(\frac{z-1}{z+1}\right)$ is purely imaginary.
What will be your conclusion if $z = 1$.
41. Find the least positive integer n if $\left(\frac{1+i}{1-i}\right)^n = 1$ (Ans: $n = 4$)
42. Find the real θ such that $\frac{3+2i\sin\theta}{1-2i\sin\theta}$ is purely real. (Ans: $\theta = n\pi, n \in \mathbb{Z}$)
43. Show that a real value of x will satisfy the equation $\frac{1-iX}{1+iX} = a-ib$ if $a^2 + b^2 = 1$ where a, b are real.
44. If x and y are different complex numbers with $|y| = 1$ find $\left|\frac{y-x}{1-\bar{x}y}\right|$ (Ans: 1)
45. Find non-zero integral solutions of $|1-i|^x = 2^x$ (Ans: no solution)
46. Find the real value of θ such that $\frac{1+i\cos\theta}{1-2i\cos\theta}$ is purely real. (Ans: $\theta = 2n\pi \pm \frac{\pi}{2}, n \in \mathbb{Z}$)
47. Find the square roots of $5 + 12i$ (Ans: $\pm(3+2i)$)
48. Find the square roots of $7 - 24i$ (Ans: $\pm(4-3i)$)
49. Find the square root of $-15 - 8i$ (Ans: $\pm(\pm(1-4i))$)
50. Find the square roots of $-5 + 12i$ (Ans: $\pm(2+3i)$)



51. Find the square root of i (Ans: $\pm \frac{1}{\sqrt{2}}(1 + i)$)

Express the following complex numbers in the polar form and hence find modulus and argument.

52. $1 + i$ (Ans: $\sqrt{2} \left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4} \right)$)

53. $\sqrt{3} - i$ (Ans: $2 \left(\cos \left(-\frac{\pi}{6} \right) + i \sin \left(-\frac{\pi}{6} \right) \right)$)

54. $-1 + \sqrt{3}i$ (Ans: $2 \left(\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3} \right)$)

55. $-\sqrt{3} - i$ (Ans: $\cos \left(-\frac{5\pi}{6} \right) + i \sin \left(-\frac{5\pi}{6} \right)$)

56. $\frac{1+i}{1-i}$ (Ans: $\cos \frac{\pi}{2} + i \sin \frac{\pi}{2}$)

57. $\frac{1}{1+i}$ (Ans: $\frac{1}{\sqrt{2}} \left(\cos \left(-\frac{\pi}{4} \right) + i \sin \left(-\frac{\pi}{4} \right) \right)$)

58. $\frac{1+2i}{1-3i}$ (Ans: $\frac{1}{\sqrt{2}} \left(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4} \right)$)

59. $\frac{i-1}{\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}}$ (Ans: $\sqrt{2} \left(\cos \frac{5\pi}{12} + i \sin \frac{5\pi}{12} \right)$)

60. $\frac{1+3i}{1-2i}$ (Ans: $\sqrt{2} \left(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4} \right)$)

61. Solve the quadratic equation $x^2 + 25 = 0$ (Ans: $x = \pm 5i$)

62. Solve the quadratic equation $25x^2 + 36 = 0$ (Ans: $x = \pm \frac{6}{5}i$)

63. Solve the quadratic equation $x^2 + x + 1 = 0$ (Ans: $x = -\frac{1}{2} \pm i \frac{\sqrt{3}}{2}$)

64. Solve the quadratic equation $x^2 - 4x + 3 = 0$ (Ans: $x = 2 \pm 3i$)